Some Implications of the Curie Symmetry Principle in Quantum Physics

John E. Gray and Allen D. Parks
Quantum Processing Group
Code B-10
Naval Surface Warfare Center Dahlgren Division
Dahlgren, VA
22448

Abstract

We discuss the concept of symmetry and its importance to understanding evolutionary processes that arise in dynamics. Using Curie's principle that the cause must be at least as symmetric as the effect, along with Rosen's generalizations, we arrive at a number of interesting results. These principles prove very useful in classifying quantum machine behavior as well as characterizing their entropy. The implications of this principle are also discussed relative to the problems of machine intelligence, symbolization of physical processes, and algorithms as transformations on symbols.

1 Introduction: The Fundamental Role of Symmetry

[1]Symmetry, in the guise of group theory, has played a fundamental role in physics and science for the last hundred years. In physics, it has played a particularly significant role in the theoretical foundations of physics; a general survey of physical applications of symmetry is Wigner [24]. Symmetry can be readily expressed in mathematical terms. Thus it is possible to recognize new realms of application for symmetry arguments [20], [7]. An example of symmetry is exhibited by a one-to-one correspondence between the members of a collection of objects that preserves some particular

[1] This paper is dedicated to Charles Ehresmann on the occasion of the 100th anniversary of his birth.

property or properties associated with the collection. For example, rotate an n-gon about its center through a $360/n$ degree angle. Each point of the n-gon other than its center is transformed into a point $360/n$ degrees away. This rotation, which preserves the shape of the n-gon, is a symmetry of the n-gon. A square, for example, has a total of eight symmetries: rotations through $90°$, $180°$, or $270°$ degrees; reflection across one diagonal; a reflection followed by rotations through $90°$, $180°$, or $270°$ and the identity transformation which corresponds to a $360°$ rotation about the center.

In general, a symmetry is a 1-1 correspondence between elements in a set which preserve some property associated with the set. All such symmetries along with their bijective inverses and the identity map forms a group under composition of maps. Thus, if f and g are symmetries taking points x and y to points $f(x)$ and $g(y)$, respectively, then composition of f and g ($f \circ g$) is the transformation taking x to $f(g(x))$ which is also a transformation symmetry. Furthermore, the inverse of symmetry f, which is the transformation taking the point $f(x)$ to the point x, is also a symmetry. The group of such maps is commonly called the **symmetry group** for the set. Symmetry has usually been associated with geometry (size and shape), but need not be since other types of properties can be preserved by bijective maps: e.g., if a and b are siblings, then so are b and a. Therefore the transformation taking a to b and b to a preserves this form of kinship. Indeed, any bijective map between the members of a family which preserves the sibling relationship can be viewed as a "sibling symmetry".

In the sciences, symmetry arguments are often used to deduce equations underlying a theory. A symmetry is proposed, the equations preserved by it – or compatible with it in an appropriate sense– are identified as candidates. Additional symmetries may further narrow the field of candidates. In art, symmetry can be linked to the aesthetic nature of harmony and composition. It can be argued that the same applies to sciences, e. g. if an equation or physical principle is considered beautiful, then probably an underlying symmetry is the source of the appeal (*Dirac's principle of aesthetics*). Abstraction of some principle from the natural world, such as symmetry, is central to the exploration of properties of symbols as well as the machines that manipulate the symbols. The nature of the symbolization process is important in understanding the physical world, but is not widely recognized as being so. Symbolization is an aspect of understanding the physics of computation and generalizations of physical models of computation. Symmetry plays an important role in symbolization.

The ability to abstract the world within language is perhaps the most unique tool human beings have; it has enabled us to construct new universes and describes the world around us. The origins of mathematics are lost in antiquity. However, it is related to some "sense of number" and enumeration (one-to-one correspondence between a marker system and a collection of objects) that most higher animals seem to posses. Most mathematicians agree that mathematics is not culture specific. Any sufficiently advanced civilization has developed a mathematical system based on three specific aspects of mathematics: **numbers** (accounting at a minimum, number theory), **measurement** (measure, weight, length, and ultimately some form of geometry), and **abstract rule based systems** (algebras or algorithms of some type). The issue of science is more problematic: it requires the ability to make comparisons–either by measurement or reduction to numbers. Thus, instrumentation plays a fundamental role. The use of scientific instruments allows one to deconstruct reality in terms of measurements that are based on some type of comparison. These instruments, while not typically thought to be abstractions, are ultimately the symbolizations that we associate with "reduction to number" and are based on relativity few mathematical concepts. Primary among these are notions of proportion, angle, length, similarity, correspondence, greater than, equality, and number. No scientific instrument functions without usage of a variety constructs based on strings of analogies using these concepts. An understanding of the abstractions associated with instrumentation plays a fundamental role in the limits of the imagination.

The essence of the physics model of reality has been to compute with numbers. However, numbers don't always capture the essence of reality; instead a better thread is achieved with words, perceptions, and models [26]. This realization is what motivated Zadeh to propose "fuzzy logic". Some applications require "computing with words" rather than computing with numbers ([26], [27], [28], [29], [30]). A measurement process using numbers and associated with perceptions by an individual or a poorly performing instrument typically lack the clarity to capture an attribute. A less crisp concept such as a "word descriptions" often works better in such cases. The symbolic process that reduces an observation to a number amounts to imposing the algebraic structure upon an observation. By saying that an observation does not lead to the crisp concept of a number, implies that the algebra associated with the measurement process has less structure than the symbolic algebra associated with number. This has important consequences that were first noted by Curie.

One hundred years ago Pierre Curie ([2],[18]) wrote a paper on symmetry in physics that can be efficiently encoded as the koan "*the effect is at least as symmetric as the cause*". The implications of this principle and its extensions are of particular interest. In particular, we are interested in how this notion of symmetry effects the behavior of finite state machines, as well as the implications of behavior on the understanding of intelligence. Based on the original Currie principle, Rosen [20] has expanded Currie's notions to argue that the relationship of symmetry to physics can be expressed in terms of six core symmetry principles. Two such aspects of symmetry are of particular interest, namely Rosen's **Symmetry Principle for Processes**, which states: *the "initial" symmetry group (that of the cause) is a subgroup of the "final" symmetry group (that of the effect),* and his **Special Symmetry Evolution Principle**, which states: *as a qausi-isolated system evolves, the populations of the equivalence subspaces (equivalence classes) of the sequence of the states through which it passes cannot decrease, but either remain constant or increase.* These principles have been used implicitly in physics over the years, but largely without recognition of their importance–particularly with reference to quantum machine operations and their implications, as well as the importance of symbolization to symbolization and how it effects computation. The most straightforward approach to understanding Rosen's symmetry principles is to first examine cyclic quantum state machines.

2 Finite Cyclic Quantum State Machines and Their Symmetries

2.1 Introduction to FCQSMs

The notion of a quantum state machine (QSM) was first introduced by Gudder [9] who took the traditional formalism of state and transition function and modified them appropriately for quantum mechanical systems based on work by [15]. Gudder's QSM is a simple quantum system which has no inputs or outputs and evolves from one state to another in simple equally spaced time steps. Parks [16] wrote a paper on finite cyclic quantum state machines (FCQSM), which are a further simplification of QSM that they evolve in a finite dimensional Hilbert space and return to their initial state after a finite number of steps. Parks explored a number of properties of such machines. and was able to show that FCQSMs always exhibit symmetries consistent with Curie's Principle and Rosen's Symmetry Evolution Principle, but are often inconsistent with Rosen's Symmetry Principle for Processes. These symmetry preserving and breaking properties exhibited by FCQSMs have some interesting implications about the nature of simulation that relate to intelligence, and also raise some interesting philosophical questions.

We first explore the nature of the symmetries associated with FCQSM. Recall that a quantum system, with normalized states $|\Psi\rangle$, belong to a $(n+1)$-dimensional Hilbert space $\mathbf{C}^{n+1}$ ($\mathbf{C}$ is set of complex numbers with $n \geq 1$) defined by the set $\mathcal{H} = \left\{|\Psi\rangle \in \mathbf{C}^{n+1} : \langle \Psi, \Psi\rangle = 1\right\}$. The set of states $\mathcal{H}$ is homeomorphic to the unit sphere S^{2n+1}, i.e. there exists a homeomorphism f such that $f : \mathcal{H} \rightarrow S^{2n+1}$. Also, the sequential application of $\hat{U}(\Delta t)$ m times to $|\Psi_0\rangle$ produces a closed cycle of length m in $\mathcal{H}$ which corresponds to the dynamic evolution of a FCQSM in $\mathcal{H}$:

$$|\Psi_0\rangle \overset{\hat{U}}{\Rightarrow} |\Psi_1\rangle \overset{\hat{U}}{\Rightarrow} |\Psi_2\rangle \overset{\hat{U}}{\Rightarrow} .... \overset{\hat{U}}{\Rightarrow} |\Psi_{m-1}\rangle \overset{\hat{U}}{\Rightarrow} |\Psi_m\rangle = |\Psi_0\rangle . \tag{1}$$

Thus, for a given integer $k \geq 1$, we have the state evolution rule:

$$\hat{U}^k(\Delta t)\,|\Psi_0\rangle = \hat{U}^{k \bmod m}(\Delta t)\,|\Psi_0\rangle = |\Psi_{k \bmod m}\rangle \tag{2}$$

which implies that the group $\left\langle \hat{U}(\Delta t)\right\rangle$ generated by $\hat{U}(\Delta t)$ is isomorphic to the finite cyclic group $\mathbb{Z}_m$ of order m. There are two maps that define the dynamics of the FCQSM: the map $\theta : \left\langle \hat{U}(\Delta t)\right\rangle \times \mathcal{H} \rightarrow \mathcal{H}$ defined by $\theta\left(\hat{U}^k(\Delta t), |\Psi\rangle\right) = \hat{U}^k(\Delta t)\,|\Psi\rangle$ for $1 \leq k \leq m$ describes the dynamics of the FCQSM as well as the map $\varphi = f \circ \theta \circ (\alpha \times f)^{-1}$ which provides the communative diagram

$$\begin{array}{ccc} \left\langle \hat{U}(\Delta t)\right\rangle \times \mathcal{H} & \overset{\theta}{\rightarrow} & \mathcal{H} \\ \downarrow (\alpha \times f) & & \downarrow f \\ \mathbb{Z}_m \times S^{2n+1} & \overset{\varphi}{\rightarrow} & S^{2n+1} \end{array} \qquad \text{(Group \& State Relabeling Diagram)}$$

Here $\alpha : \left\langle \hat{U}(\Delta t)\right\rangle \rightarrow \mathbb{Z}_m$ is the group isomorphism mentioned above. Thus, the two maps relabel group elements and states in a manner that preserve the group properties of $\left\langle \hat{U}(\Delta t)\right\rangle$ and the topological properties of $\mathcal{H}$. The characteristics of the Group & State Relabeling Diagram allow us to characterize the dynamics of FCQSMs using the properties of φ. One can then refer to all machines as (m, n) −FCQSM where m refers to the order of $\mathbb{Z}_m$ and n refers to the dimension of S^{2n+1} and the machine is said to "be defined by $\mathbb{Z}_m$". Note that the $(m, 1)$ −FCQSMs are *qubit machines*–finite cyclic processes in two dimensional Hilbert spaces.

It is shown in [16] that the map φ defines a continuous free left $\mathbb{Z}_m$−action on S^{2n+1}. As a consequence of this there exists an induced canonical projection $p : S^{2n+1}/\mathbb{Z}_m$ which is also a universal covering . Here, $S^{2n+1}/\mathbb{Z}_m$ is the quotient space generalized by the action φ. Each point x in $S^{2n+1}/\mathbb{Z}_m$ is a process cycle and each fiber $p^{-1}(x)$ is the set of states in the cycle.

Definition: A (m,n) −FCQSM simulates an (m',n) −FCQSM if there exists a surjective map $r : S^{2n+1}/\mathbb{Z}_{m'} \to S^{2n+1}/\mathbb{Z}_m$ such that the diagram

$$\begin{array}{ccc} & S^{2n+1} & \\ {\scriptstyle q}\swarrow & & \searrow{\scriptstyle p} \\ S^{2n+1}/\mathbb{Z}_{m'} & \stackrel{r}{\rightarrow} & S^{2n+1}/\mathbb{Z}_m \end{array} \qquad \text{(FCQSM Simulation Diagram)}$$

commutes.

The usefulness of this concept of simulation is due to the following theorem (see [16] for proof). It allows one to consider the concept of self-simulation via subgroups of $\mathbb{Z}_m$.

Self Simulation Theorem (SST): Let $\mathcal{M}$ be a FCQSM defined by $\mathbb{Z}_m$. For every non-trivial subgroup of $\mathbb{Z}_m$, there is a FCQSM that is simulated by $\mathcal{M}$.

Such simulations are called $(m,n)\,/\,(m',n)$ *simulations* and $\mathcal{M}$ is said to be (m',n) *capable*. The meaning of the $(m,n)\,/\,(m',n)$ *simulation* is that [16]:

> "the states of the process cycles for (m,n) −FCQSMs 'geometrically register' the states of the process cycles for (m',n) −FCQSMs such that each (m,n) −FCQSM cycle 'registers' m/m' (m',n) −FCQSM cycles."

This leads to the following definition and theorem:

Definition (simulation ratio): The simulation ratio σ is defined as the group index

$$\sigma = [\mathbb{Z}_m : \mathbb{Z}_{m'}] \equiv \frac{|\mathbb{Z}_m|}{|\mathbb{Z}_{m'}|} = \frac{m}{m'} \tag{3}$$

where $|G|$ is the order of the group G (which implies the following theorem).

Short Exact Sequence Theorem: For every $(m,n)\,/\,(m',n)$ *simulation* for which $m \neq m'$, there exist short exact sequence

$$1 \to \mathbb{Z}_{m'} \stackrel{\iota}{\to} \mathbb{Z}_m \stackrel{\vartheta}{\to} \mathbb{Z}_\sigma \to 1 \qquad \text{(Short Exact Sequence)}$$

where ι is an injective homorphism and ϑ is a surmorphism ((see [16] for proof)).

One observes that the quotient group $\mathbb{Z}_m/\mathbb{Z}_{m'} \approx \mathbb{Z}_\sigma$ in the short exact sequence defines a (σ,n) −FCQSM quotient machine when $m \neq m'$. A

$(m,n)\,/\,(m',n)-$*simulation* for which $m \neq m'$ splits if its short exact sequence splits, i.e. when $\mathbb{Z}_m \approx \mathbb{Z}_\sigma \oplus \mathbb{Z}_{m'}$ (here $\approx$ "is isomorphic to" and $\oplus$ denotes "direct sum of Abelian groups"), and an $(m,n)-$FCQSM which exhibits a split simulation is said to a split FCQSM. It may therefore be concluded from the following commutative diagram that "every split FCQSM simulates an induced $\sigma-$machine":

$$\begin{array}{ccccc} & & S^{2n+1} & & \\ & \overset{q}{\swarrow} & \downarrow^{p} & & \overset{s}{\searrow} \\ S^{2n+1}/\mathbb{Z}_{m'} & \overset{r}{\rightarrow} & S^{2n+1}/\mathbb{Z}_m & \overset{t}{\leftarrow} & S^{2n+1}/\mathbb{Z}_\sigma \end{array}$$

(Split Simulation Diagram)

Similar more complicated machine diagrams exist for any subset of machines simulated by a FCQSM.

2.2 Classification of symmetries

Since the dynamics of an $(m,n)-$FCQSM are represented by the group $\mathbb{Z}_m$ and the global processing associated with these dynamics is represented by the covering $p : S^{2n+1} \to S^{2n+1}/\mathbb{Z}_m$, then two types of symmetry can be identified with the algebraic properties of machines: dynamic symmetries (automorphism of $\mathbb{Z}_m$) and process symmetries (covering space automorphism). An automorphism for the covering $p : S^{2n+1} \to S^{2n+1}/\mathbb{Z}_m$ is a homeomorphism $\eta : S^{2n+1} \to S^{2n+1}$ such that the diagram

$$\begin{array}{ccc} S^{2n+1} & \overset{\eta}{\rightarrow} & S^{2n+1} \\ \underset{p}{\searrow} & & \underset{p}{\swarrow} \\ & S^{2n+1}/\mathbb{Z}_m & \end{array}$$

(FCQSM Curie's Simulation Principle)

commutes. Note that each such η determines an immunity to change in the topological structure of the covering, and therefore represents a distinct symmetry for the covering. The set of all such homorphisims under composition is the group of processes symmetries $Cov\,(p)$ for the associated FCQSM and $|Cov\,(p)|$ is its order. Since $Cov\,(p) \approx \mathbb{Z}_m$, it follows that every FCQSM simulates its group of process symmetries.

A group automorphism for $\mathbb{Z}_m$ is an isomorphism $\beta : \mathbb{Z}_m \to \mathbb{Z}_m$. The set of all such isomorphisms under composition is the group of dynamical symmetries $Aut\,(\mathbb{Z}_m)$ for the machine and $|Aut\,(\mathbb{Z}_m)|$ is the order. A FCQSM is said to be dynamically trivial if $|Aut\,(\mathbb{Z}_m)| = 1$, i.e. when $m = 2$. FCQSMs have two types of symmetry principles that are relevant, a strong symmetry principle which satisfies Rosen's Principle for Symmetry Processes and a weak version adheres to the Currie principle. The **weak FCQSM symmetry principle** asserts for all FCQSMs: "*a process is more symmetric than the dynamics which performs it.*" This can be interpreted as a statement about the relative cardinalities of the distinct sets of dynamic and process symmetries and follows from the fact that $|Aut(\mathbb{Z}_m)| < |Cov(p)|$ (see [16]). The **strong FCQSM symmetry principle** asserts that for all strongly symmetric FCQSMs: "*the group of dynamical symmetries is isomorphic to a subgroup of the group of process symmetries.*" This strong version is a symmetry conservation principle for *FCQSMs* which requires that for a strongly symmetric machine a faithful copy of the group of dynamic symmetries is always contained within its group of process symmetries.

Using special machines with simulation ratios given by $\frac{m}{\phi(m)}$, where ϕ is the Euler totient and the $(\phi(m), n)-$FCQSM is referred to as the $\phi-$machine, one can deduce several classification theorems:

1. A dynamically non-trivial strongly symmetric FCQSM simulates its associated $\phi-$machine.

2. If a dynamically non-trivial FCQSM is not capable of simulating its associated $\phi-$machine, then it is strictly weak (FCQSM are strictly weak when it is not strongly symmetric).

3. Every $(m, n)-$FCQSM for which m is odd is strictly weak.

It is shown in [16] that while every FCQSM is weakly symmetric, not all FCQSM's are strongly symmetric. Thus, we have the implication that

strong FCQSM symmetry principle $\Rightarrow$ **weak FCQSM symmetry principle**

is valid but the converse isn't. So that the set $\mathcal{S}$ of machines that satisfy the strong principle are proper subsets of the set $\mathcal{W}$ of all FCQSM's. Within this context, such weak FCQSM's in the set $\mathcal{W}-\mathcal{S}$ violate Rosen's Symmetry Principle for Processes. Furthermore, from [16] we find:

Theorem 1 *If a FCQSM adheres to Rosen's Symmetry Principle for processes, then m is even and $Aut(Z_m)$ is cyclic.*

Thus, one may classify FCQSMs according to whether they are in set $\mathcal{S}$ or in set $\mathcal{W}-\mathcal{S}$. We pose the question "*does this symmetry based dichotomy indicate the existence of a fundamental difference between these machine classes?*" This question is difficult in general to answer. However, we can provide further insight into the answer to the question by quantifying aspects of these principles using the notions of efficiency and entropy.

2.3 Efficiency and Entropy

In the language of categories, the homotopy functor can be employed to define an *induced topological complexity index* $\Gamma_{m,n}$ associated with FCQSM processing. In particular, this index can be defined as [16]

$$\begin{aligned}\Gamma_{m,n} &= \left|\Pi_1\left(S^{2n+1}/\mathbb{Z}_m\right) - \Pi_1\left(S^{2n+1}\right)\right| \\ &= |\mathbb{Z}_m| - 1 \\ &= m-1 \end{aligned} \tag{4}$$

where $\Pi_1(X)$ is the fundamental group for the topological space X. Note that $\Gamma_{m,n}$ measures differences in the number of one dimensional holes found in the quotient space and S^{2n+1} (i.e., between the "processed" space $S^{2n+1}/\mathbb{Z}_m$ and the "unprocessed" space S^{2n+1}). But $|Aut\,(Z_m)| = \varphi(m)$, where φ the Euler totient function [25]. This is the number of dynamic symmetries for a FCQSM of order m. Also, since $\varphi(m) \leq m-1 = \Gamma_{m,n}$, then

$$|Aut\,(Z_m)| \leq \Gamma_{m,n} \tag{5}$$

which states that the number of dynamic symmetries for a FCQSM can never exceed the increase the number of one dimensional holes induced by its processing. Note, there are two possibilities for m:

Case 1: When m is a prime, then $\varphi(m) = m-1 \Rrightarrow |Aut\,(Z_m)| = \Gamma_{m,n}$.

Case 2: When m is not a prime, then $\varphi(m) \leq m-1 \Rrightarrow |Aut\,(Z_m)| \leq \Gamma_{m,n}$.

This can be interpreted as saying something about the "dynamic efficiency" of a machine in terms of processing induced holes. Specifically, if dynamic symmetries are viewed as "tokens" which are "spent" producing holes during processing, then FCQSM's can be classified according to

> "a prime FCQSM (for which m is a prime number) uses all of its tokens during processing to make the holes, whereas a non-prime FCQSM (for which m is not a prime number) doesn't, i.e. non-prime FCQSM's are more efficient than prime FCQSM's."

Thus we can say, that –excluding the prime machines of order $m = 2$- even machines of order $m' = 2m$ are more efficient than machines of order m. To see this, let the distinct prime divisors of m be p_1,p_2,...,p_k. Since

$$\varphi(m) = m\left(1 - \frac{1}{p_1}\right) \ldots \left(1 - \frac{1}{p_k}\right) \tag{6}$$

and

$$\begin{aligned}\varphi(m') &= m'\left(1-\frac{1}{2}\right)\left(1-\frac{1}{p_1}\right)...\left(1-\frac{1}{p_k}\right) \\ &= \frac{m'}{2}\left(1-\frac{1}{p_1}\right)...\left(1-\frac{1}{p_k}\right) \\ &= m\left(1-\frac{1}{p_1}\right)...\left(1-\frac{1}{p_k}\right),\end{aligned}$$

then

$$\varphi(m) = \varphi(m'). \tag{7}$$

The m' machine produces $2m-1$ holes in the quotient space using $\varphi(m)$ tokens, whereas the m machines produces $m-1$ holes using the same number of tokens. Thus $m'-$machines are more efficient than the $m-$machine with its token usage. An amusing way to think about this is to imagine them as Pacmen that consume spheres and produce from them new topological spaces with even more holes than the sphere. Each hole has a cost requiring the expenditure of tokens by a FCQSM Pacman. The efficiency of the process of consumption is measured by the number of tokens it takes to produce a hole. The fewer the tokens, the more efficient the machine, the more tokens it takes, the less efficient the machine.

We can relate efficiency to the concept of entropy as well by using the definition of thermodynamic efficiency [6]. If we denote the least work rate required by an actual task as the rate $\dot{W}^{\Leftarrow}_{\text{least}}$ and the actual energy source consumption rate $\dot{W}^{\Leftarrow}_{\text{most}}$, then the *thermodynamic efficiency* or *effectiveness* ϵ is

$$\epsilon = \frac{\dot{W}^{\Leftarrow}_{\text{least}}}{\dot{W}^{\Leftarrow}_{\text{most}}} \tag{8}$$

The effectiveness measures the degree to which processes being carried out are reversible ($\epsilon = 1$) or irreversible ($\epsilon < 1$). Effectiveness can also be expressed as

$$\epsilon = 1 - \frac{T_R\dot{S}_{\text{irr}}}{\dot{W}^{\Leftarrow}_{\text{most}}} \tag{9}$$

where $\dot{S}_{\text{irr}}$ is the total rate of entropy generation by an irreversible source and $T_R\dot{S}_{\text{irr}}$ is the lost work rate. Thus we can define the entropy in terms of the efficiency as

$$T_R\dot{S}_{\text{irr}} = (1-\epsilon)\,\dot{W}^{\Leftarrow}_{\text{most}}. \tag{10}$$

For the Pacman eating the sphere, make the replacement

$$\dot{W}^{\Leftarrow}_{\text{least}} \rightarrow \dot{W}^{\text{token}}_{\text{least}} = \varphi(m), \tag{11}$$

and

$$\dot{W}_{\mathrm{most}}^{\Leftarrow} \rightarrow \dot{W}_{\mathrm{most}}^{\mathrm{token}} = m - 1, \tag{12}$$

so the efficiency is

$$\epsilon(m) = \frac{\varphi(m)}{m-1}. \tag{13}$$

Now since we are discussing FCQSM's, define $T_R \dot{S}_{\mathrm{irr}} = S^{\mathrm{token}}(m)$ so that the token entropy is

$$S^{\mathrm{token}}(m) = m - 1 - \varphi(m). \tag{14}$$

The definition of token entropy and efficiency are consistent with our discussion from above.

Two observations about this entropy can be made from this expression for the entropy given the properties of the Euler totient function:

1. The entropy is always greater than or equal to zero (it is zero when $\varphi(m) = m - 1$, e.g. m prime.

2. Since, for $m \neq 2, 6$, $\varphi(m) \geq \sqrt{m}$ and $m - \sqrt{m} \geq \varphi(m)$, then

$$S^{\mathrm{token}}(m) \geq \sqrt{m}. \tag{15}$$

   so the entropy growth is a function for all machines with $m \neq 6$.

Some additional comments can be made about the token entropy that are related to the symmetry evolution principle. Rosen has noted the correspondence

$$\text{degree of symmetry} \Leftrightarrow \text{entropy} \tag{16}$$

which suggests that there should be an equivalence between the symmetry evolution principle and the second law of thermodynamics. Within the context of dynamical symmetries and their number, what we have shown above is consistent with the more general result that a fundamental relation exists between entropy and symmetry.

2.4 Implications for Machine Intelligence

We suggest that the ability of a system to internally simulate aspects of its internal dynamics has implications for machine intelligence (internal simulation of external behavior of the world being a form of intelligence) as well as possibly for other forms of intelligence. One observation related to entropy is that the FCQSM's of prime order have zero entropy and non-prime machines have positive entropy. Included in this class of machines with positive entropy are dynamically non-trivial strongly symmetric machines (Theorem 1 above) which can simulate their dynamic symmetries. Perhaps the associated non-zero entropy is necessary condition that must be satisfied in order for systems to construct internal dynamical simulations (and other forms of self knowledge). The ability to predict dynamical behavior is centrally important to survival in the real world (e. g. kill it, eat it, rob it, or have sex with it; the usual human preoccupations).

If an agent can model its environment (construct an internal simulation), then it can exploit it to its advantage. This is almost a principle of ontology that explains some aspects Darwin's survival of the fittest by removing the tautology. It can also be deduced that successful models that enable one to predict have the following attributes:

1. An internal model for an agent must capture some significant (statistically significant effect on probability of survival) aspect of the external world.

2. The processing speed for the internal model must synchronize with the external behavior of the world (clock symmetry) in a manner that makes prediction meaningful and useful.

3. The model of the external world's environmental features must have a finite length description that has a useful approximation in raw form or is compressible to a form that is storable.

4. The reward for expending effort to model the external world exceeds the effort required to accomplish it.

5. Change occurs on a time scale that makes agent modeling useful.

6. Abstraction to finite symbols to model the external world can be accomplished at a reasonable error transcription rate.

7. The agent's model correction mechanism exists and is effective.

Because the types of machines we have talked about are so simple, we have made an inductive leap in asserting more general properties of "intelligent behavior" for agents and perhaps needs further justification. To more fully discuss these matters, we consider machines that manipulate symbols in light of what the impact of what we have learned from the symmetry properties of FCQSMs.

3 Symbolization

As part of the process of abstraction, we don't have to deal with numbers. Instead we model process as collections of symbols. With structures which are interpreted as words, we can produce models which employ transformational process on symbols rather than numerical procedures employing numbers. These modeling structures resemble that of an automata ([10], [11], [17]) so "computation" can be considered to be processes that engage in translations of one word into another. Note computation can always be cast in terms of symbolization. Classical computability theory is structured around the formal notion of a classical Turing machine in terms of effective processes, i.e. those processes which can be performed in a determinate and precisely specified manner using steps which can only be executed by finite mechanical means. Thus, in order that a process be effective, it must possess the following properties: **mechanistic**- it consists of a finite sequence of instructions each of which can be carried out without insight, ingenuity or guesswork; **deterministic**- when presented with an input string, it always produces the same result. Besides the Turing machine, one can discuss computation in terms of algorithms that operate directly on symbols. With a viewpoint of the symbolic in mind, computation can be viewed as the execution of an algorithm. An algorithm can be viewed as a finite set of rules that solve a specific type of problem. Informally, there are five important features of an algorithm that characterize it completely [11]:

1. **Finiteness:** The algorithm must terminate after a finite number of steps. (A procedure that has all the other characteristics of an algorithm except finiteness is a *computational method*.)

2. **Definiteness:** Each step must be precisely defined, so actions can be both unambiguously and rigorously specified. (This implies the requirement for formal programing languages that are designed so that each step in an algorithm has a definite meaning. An execution of an algorithm in a programing language is termed a *program*.)

3. **Inputs:** An algorithm has zero or more quantities that are provided to it before the algorithm starts execution that are taken from a specified set of objects.

4. **Outputs:** An algorithm has quantities termed outputs that have a specified relationship to the inputs to the algorithm *that are also to be taken from the same collection of objects responsible for the inputs.*

5. **Effectiveness:** An algorithm should be sufficiently basic that all its operations can be carried out exactly in a finite length of time by someone using a pen and paper. (An algorithm cannot be stated in terms of a property that is not already algorithmic or in terms of non-defined objects, infinite decimals or a typical cooking book recipe for example.)

While these capture what naturally meant by an algorithm, one can adapt a more formal specification of an algorithm based on the model of algorithms proposed by Markov [11]. One must also have a notion of the problem formulated in the proper language relative to the symbol set.

Any notion of what constitutes a problem starts with a formulation that is expressed in a particular language. Being a language, it is by necessity an expression of a sequence of symbols within that language (with the understanding that a blank, which is used to separate words is a symbol in its own right.) An **alphabet** ¥ is a non-empty finite collection of words. We also require that sequences formed from the alphabet be a denumerable sequence. One can then define a **word** in ¥ to be any finite sequence of symbols in ¥. (One also needs an empty word Υ.) For a word P that is denoted $S_{j_1}...S_{j_k}$ and a word Q denoted $S_{r_1}...S_{r_m}$, the juxtaposition PQ is $S_{j_1}...S_{j_k}S_{r_1}...S_{r_m}$ of the two words. Note that juxtaposition has the properties $P\Upsilon = \Upsilon P = P$, and $(P_1P_2)P_3 = P_1(P_2P_3)$. An alphabet A is an **extension** of an alphabet B iff $B \subseteq A$. If an alphabet is an extension of another alphabet, then any word in the second alphabet is in the extension of the alphabet. An **algorithm in** an alphabet A is a computation £ that has the properties previously described whose domain is a sub-collection of the words of A and values are also in A. If P is a word in A, $\pounds$ is **applicable** to P if P is in the domain of $\pounds$; if $\pounds$ is applicable to P, we denote its value by $\pounds(P)$. An **algorithm over** an alphabet A means that $\pounds$ is in an extension B of A. One can then use a single operation, substitution of one word for another as the basis for construction of all algorithms. These simple definitions provide all that is necessary to construct a theory of computation where the algorithms are rules for transformation of one word into another.

4 Implications of Rosen's Principle for Symbolization

Given a dynamic model, it is always possible to arrive at a symbolic model that is entirely equivalent to the dynamic model ([3] and [13]) by using a sensor as the (symbolic) transformation device that translates measurements into a finite alphabet which spans the space of possible descriptions. (Note that this observation is implicit in all of Shannon's work on information theory [21].) Using a finite alphabet of N symbols, one can define a 'symbolic space' associated with a dynamics model. For our purposes, the elements of $\sum_N$ are all finite strings of symbols. (Note in other applications the strings can be infinite.) A process that takes measurements and maps them into observations, e.g. the mapping transforms of analog sensor information into symbols that can be concatenated into strings drawn from a model based symbol set. The reduction to a symbol alphabet has symmetry issues [8] that can be associated with either dynamic or process symmetries:

1. Model Typing (process symmetry): How many symbols are sufficient to characterize the measurements?

2. Model Association (both): How does one associate a measurement vector $|M\rangle$ with the symbol set?

3. Model Genotype (dynamic symmetry): A genotype for a measurement is an assignment or association of the measurements $|M\rangle$ to the vector of possible model types

$$[\lambda_1^x x_1, \lambda_2^x x_2, ..., \lambda_n^x x_n, \lambda_1^y y_1, \lambda_2^y y_2, ..., \lambda_n^y y_n, \lambda_1^z z_1, \lambda_2^z z_2, ..., \lambda_n^z z_n] \quad (17)$$

   where the $\lambda_i^{()\prime}s$ represent the degree of belief that the symbol is representative of measurement. Note, by exhaustion, we require

$$\sum_{i=1}^{n} \lambda_i^{()} = 1 \quad (18)$$

   For convenience, it is useful to adjoin the additional symbol 0 to the set so we can deal simply with the case $\lambda_j^{()} = 0$.

4. Model Clarity (process symmetry): A model has clarity if we have a high degree of confidence that a single symbol represents it, while it is maximally unclear if $\lambda_j^{()} = \frac{1}{n}$ where n is the number of symbols.

5. Model Age (dynamic symmetry): The age of a genotype is the number of data updates k that have lead to the present genotype, for example this would be represented in a genotype as:

$$[\lambda_1^x x_1(k), \lambda_1^y y_1(k), \lambda_1^z z_1(k)] \quad (19)$$

6. Transcription Accuracy (Noise Effects) (both): The effect of noise is to determines the model algebra behavior.

7. Model Algebras (both): There are several different algebras associated with the dynamics of mapping of the data into model type. By algebra we mean that there are mappings associated with the data transcription to model type (*measurement algebra*), the algebra associated with updating a genotype from one update to the next (*transition algebra*), and if the genotype has limited clarity, only a sub-algebra of the model symbols may be in play rather than all of them for small age differences (*clarity algebra*). The subgroups of the permutation group are a means for discussing these models.

8. Model Typing or Identification (process symmetry): If we have a series of model genotypes and have been able to infer a model algebra associated with that genotype, then an additional inference can be formed that allows one assign an additional symbol drawn from a new symbol set [19]. This typing symbol set is a higher level alphabet that is largely non-dynamic. It has the specific characteristics of what we might term– as drawn from biological nomenclature– a species. Drawing the inference of a species from model typing sources can be also termed identification.

The measurement process lends itself to the speciation of the continuum by reducing all possible measurements to a finite symbol alphabet. The simplest alphabet, which is used in classical logic gates, is based on voltage signals which are reduced to the symbolic level by a measurement/conversion process to the symbols 1 and 0. For example, the simplest case of a universal symbolization is the conversion of an analog signal into a digital signal set, specifically the analog-to-digital (A-to-D) converter [14]. Formally, an A-to-D converter takes a continuous real time signal $t \longmapsto x(t)$ $(t \geq 0)$ and generates an output sequence $\{\widehat{x}_k : k \in Z^+\}$ from a finite alphabet J. The A-to-D converter produces a symbol $\widehat{x}_k$ at a time $k\widehat{\tau}$, where $\widehat{\tau}$ is a prescribed sampling interval for $\tau > 0$. Note, this process is inherently nonlinear as are most symbolic conversion processes, though this is not widely acknowledged. Thus a digital symbol set consists of the two symbol alphabet $\{0, 1\}$ so that $\sum_2$ is in the set of all finite strings of $0's$ and $1's$. When dealing with communication or digital encoding, words or messages consist of long strings of ones and zeros which are elements in $\sum_2$.

Without numbers, we do not have a precise grammar which can be interpreted additional degrees of freedom (in physics parlance). This allows multiple deconstruction in terms of symbolic models rather than a single deconstruction. Reduction of signals to symbols exhibit a variety of symmetries that are associated with the process. Because the alphabet of symbols is finite, they represent classes that have a degree of invariance because of the inability to distinguish between them (an example of this is translational invariance). At least one form of invariance that is necessarily required for any type of symbolization is the identity. Also, one can define an equivalence relation upon a set of signals using a mapping S from signals v onto symbols, i.e.

$$v- > S(v). \tag{20}$$

If u and v are arbitrary signals, then one can write $u \sim v$ if $S(u) = S(v)$. With an alphabet, there is an inherent uncertainty in the assignment of signal to symbol which can be symbolized as $P(s/v)$ which is the probability that the symbol s is the correct representation of some part of the signal space spanned by a signal set provided the corresponding signal is v. A further explanation for the symbolic basis for measurement [23].

5 Conclusions

Curie's principle has appeared in a number of unexpected regimes. Physics based modeling indicates that there are a variety of algebraic considerations that arise from symmetry considerations which suggest that there are physics based algebras awaiting our discovery. The interesting thing about these physical algebras is that they could motivate new forms of syntax as well as semantics. Thus, as practitioners of measurement, we can use measurement algebras as a means of providing insights into new mathematical objects, as well as new forms of syntax for use by mathematicians. By removing semantic assumptions from the objects, mathematicians can achieve clarity with regard to the syntax. This increases the rigor of usage by those engaged in the external world. Practical issues associated with the symbolization perspective force us to re-examine communication theory issues such as sampling, bandwidth, noise, and representation which arise from the symbolic rather than the numeric perspective. Issues such as require reformulation from the transformational perspective and lends itself to the discussion of the transformations of symbol sets by machines or algorithms.

There are several problematic observations that arise from consideration of the symbolic and simulation perspective that raise questions that category theory perhaps, can help answer. When converting a signal to a finite string of symbols, it is not always possible to distinguish between analog features with low probability of occurrence. Thus, some features can arise in time series data that are ambiguous with respect to symbolic characterization. We pose the following question: *Is it possible to develop categorical descriptions of ambiguity and the allied notion of entropy and study functorial relationships between these categories and the category of groups in order to provide insight into new symmetry based approaches to ambiguity reduction?*

There is a hierarchical classification of machines that is naturally induced by the grammatical properties of the associated families of languages accepted by machines. We pose the following questions: *Can functorial relationships be established between categories of machines and languages that suggest further refinements of and relationships between these classification schemes? and Can functorial relationships between a category of algorithms and the category of digraphs yield new and meaningful measures of algorithmic complexity?*

There are many insights yet to be found by examining physical models from the symmetry perspective suggested by Rosen's generalization of the Curie principle.

References

[1] Chaitin, G. J., *Algorithmic Information Theory* Cambridge Tracts in Theoretical Computer Science 1, 1987.

[2] Curie, P. , "Sur la symetrie dans les phenomenes physiques d'un champ electrique et d'un champ magnetique," *J. Phys.* (3rd ser.) **3** 393-415, 1894.

[3] Devaney, R. L., *Introduction to Dynamic Systems*, Addison-Wesley, 1987.

[4] Dyson, F. "Scientific Revolutions", *The Sun, The Genome, and The Internet*, Oxford University Press, 1999.

[5] Ekert, A. and Jozsa, "Quantum Computation and Shor's factoring algorithm", *Reviews of Modern Physics*, Vol. 68, No. 3, July 1996.

[6] Gyftopoulos, E. P. and Beretta, G. P., *Thermodynamics: Foundations and Applications*, Dover Publications, 2005.

[7] Gray, J. E. and Vogt, A. J., "Is Symmetry Informative?", BioSystems 43 (1997), pp 55-61.

[8] Gray, J. E. and Alouani, A. T., "Multiple model tracking using fuzzy clustering", SPIE Conference 5428, April 13-15, 2004, Signal and Data Processing of Small Targets 2004, Volume 5428.

[9] Gudder, S.,"Basic properties of quantum automata", *Found. Phys.*, **30**, 2000.

[10] Hopcroft, J. E. and Ullman, J. D., *Introduction to Automata Theory, Languages and Computation*, Addison Wesley, Reading MA, 1979.

[11] Knuth, D. E., *The Art of Computer Programing Volume 1, Fundamental Algorithms*, Third Edition, Addison-Wesley Press, 1997.

[12] F. W. Lawvere and S. H. Schanuel, *Conceptual Mathematics: A first introduction to categories*, Cambridge University Press, 1997.

[13] Lawson, M. V., *Finite Automata*. CRC Press, 2003.

[14] Lind, D. and Marcus, B., *An Introduction to Symbolic Dynamics and Coding*, Cambridge University Press, 1995.

[15] Moore, C. and Crutchfield, "Quantum Automata and Quantum Grammars", arXiv.quant-phy/9707031v1, 16 July, 1997.

[16] Parks, A. D. , "Finite cyclic quantum state machines: a topological perspective," J. Phys. A: Math Gen. **34** (2001) 5731-5743.

[17] Papdimitriou, C. H. *Computational Complexity*, Addison-Wesley, Reading MA, 1994.

[18] J. Rosen and P. Copie, "On symmetry in physical phenomena, symmetry of an electric field and a magnetic field," reprinted in J. Rosen (editor), *Symmetries in Physics: Selected Reprints* (American Association of Physics Teachers, Stony Book, NY, 1982.

[19] Reed, M. "Algebraic Structure of Genetic Inheritance", Bulletin of the AMS, 1997

[20] J. Rosen, *Symmetry in Science: An Introduction to the General Theory*, Springer-Verlag New York, 1995.

[21] Shannon, C. E. and W. Weaver, *The Mathematical Theory of Communication*, University of Illinois Press, Urbana, 1964.

[22] W. Wang and D. H. Johnson, "*Computing Linear Transforms of Symbolic Signals*", *IEEE Trans. Signal Processing*, March., 2002

[23] Wilson, R. and Knutsson, H., "*Uncertainty and Inference in the Visual System*", *IEEE Transactions on Systems, Man, and Cybernetics*, Vol. 18, No. 2, March/April, 1988.

[24] Wigner, E. P., *Symmetries and Reflections: Scientific Essays,* Ox Bo Press Connecticut, 1979.

[25] http://mathworld.wolfram.com/TotientFunction.html

[26] Zadeh, L.A.; "*Fuzzy logic = computing with words*", *IEEE Transactions on Fuzzy Systems*, Volume: 4 , Issue: 2 , May 1996.

[27] Zadeh, L. A., "*Toward a restructuring of the foundations of fuzzy logic*" (FL) The 1998 IEEE International Conference on Fuzzy Systems Proceedings, (1998). *IEEE World Congress on Computational Intelligence.*, Volume: 2 , 4-9 May 1998.

[28] Zadeh, L.A., *From computing with numbers to computing with words. From manipulation of measurements to manipulation of perceptions*; *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, (1999a), Volume: 46 , Issue: 1 , Jan. 1999.

[29] Zadeh, L.A, "*A new direction in fuzzy logic-toward a computational theory of perceptions*"; *Fuzzy Information Processing Society, 1999. NAFIPS. 18th International Conference of the North American*, (1999b) 10-12 June 1999.

[30] Zadeh, L.A, "*Precisiated natural language - toward a radical enlargement of the role of natural languages in information processing, decision and control.*"; *Proceedings of the 9th International Conference on Neural Information Processing*, (2002). ICONIP '02, Volume: 1, 18-22 Nov., 2002.